2.2 *Counting Outcomes*

Overview of the lesson plan:
Is it necessary to play a game many times to figure out the likelihood of winning? If not, how can we measure this likelihood? In this lesson, students learn that they can find the probability of winning a game by finding the ratio all the possible ways to win to all the possible outcomes for the game.

Daily goals:
Students will

1. Learn to calculate how many "tickets" there are by using tree diagrams to count.
2. Identify winning "tickets" on the tree diagram, understanding that there are multiple possible combinations to make a winning ticket.
3. Represent the probability as a fraction based on the tree diagram.

Summary of activities:

- **Introduction to counting outcomes and probability**: Teacher uses the single-bet roulette example in Chances Part I to introduce the concept of counting outcomes and writing probability as a fraction.
- **Creating a probability tree**: Students create a probability tree to represent all possible outcomes in the Color Pick game. Students identify and count winning "tickets" on the tree to come up with the probability of winning the game.

Mathematical ideas
This lesson introduces students to the idea of finding probability by counting outcomes. Students explore the meaning of what an outcome is and learn to create a tree diagram to count the outcomes in order to find the probability.

Making a model: In this lesson, students learn about the importance of having a systematic way to count all possible outcomes in a combination or permutation situation. The tree diagram is one specific tool that students learn to use in this lesson. The tree diagram provides a way for students to visually organize the possible outcomes so that there is less chance of missing an outcome. Some students may already have their own way of counting systematically; if so, help them make connections with the tree diagram so that they will have multiple tools for counting outcomes in the future.

Materials:

- ❑ PowerPoint for Chances 2
- ❑ Color branches and leaves in labeled bags
- ❑ Chart paper, 1 for each group
- ❑ Glue stick, 1 for each group
- ❑ Scissors, 1 for each group
- ❑ Tape, 1 for each group

Lesson Plan Outline

12:00pm	Introduction to probability (10 min)

- Do now **(Slide 2)**

Do Now	The Do Now question serves as a warm up to remind students about the previous lesson and the factors one must consider to measure probability.
Silently write your answer to the following question in your notebook: Remember yesterday's Color Combination game. Why did we have more luck winning this game than roulette or the Sweet Million lottery?	

- Go over homework – have students share their game ideas and explanations
- Introducing probability **(Slide 3)**

Class Discussion	Answers vary. This question is meant to be open-ended and encourage students to talk about what probability means and explain their sense of how 1 in 38 describes probability.
The probability of winning in single-bet roulette is 1 in 38. What does that mean? 	

- **Slide 4**

Problem	Use this as an opportunity to bring up what happened yesterday and ask Orlando to reiterate his point that if the chances were closer to 50% then there would have been more winners.
Yesterday, people said that the chances of winning the Color Pick game are 3/5. Is this right?	

12:10pm	Make a Model (10 min)

- **Slide 5**

Make a Model	• Ask students to come up with all the ways that you could have drawn a winner in the color-pick game. • Before you go to the tree model, give students a chance to think about this question. They might try to start making a list, but it gets hard to make a list in a systematic way.

Make a Model

- Begin to model the use of a tree diagram to organize possible outcomes. The idea is that rather than randomly coming up with a list of all the possible combinations, we can go about it systematically.

12:20pm	Work time (20 min)

- **Slide 7.** Give students instructions for this activity:
- Break up students into groups. Each group should get a set of color branches–one color for each of the colors used in the simulation.
- Circulate and help students as they work.
- Students get their leaves after they have arranged their branches and demonstrate that they understand what their leaves should look like.
- Take a photo of each group's branch to make a collective tree on the smart board.

12:40pm	Counting leaves (5 min)

- **Counting all leaves: Slide 8**
- [gather photos of each group's branch to create class tree diagram]

How many leaves?

[gather photos of each group's branch to create class tree diagram]

Q: Is there a way to figure this out without counting each leaf?

- Encourage them to use their tree diagrams to explain. The purpose of this is to have students look <u>across</u> all the branches and see that they are the same in structure.
- Students should see that there are 5 ways to choose the first color. Then for each of these 5 ways, you can pair them with 4 other colors, then for each of the 20 pairs, we have 3 choices for the third color, so we multiply that by 3, or 5 x 4 x 3.

12:45pm	Finding Probability (10 min)

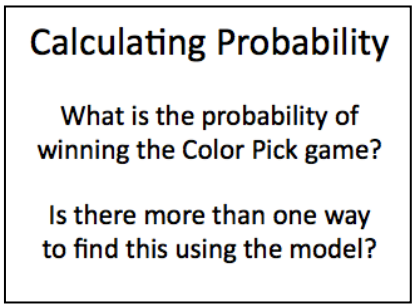
Class Problem

What is the probability of winning the Color Pick game?

Slide 9

- Choose/draw a winning combination.
- Have student bring up winning leaves to the board.
- Discuss idea that each trio can be arranged 6 different ways. In other words, each **combination** can be arranged 6 different ways.
- Now let's explore how many distinct combinations there are.

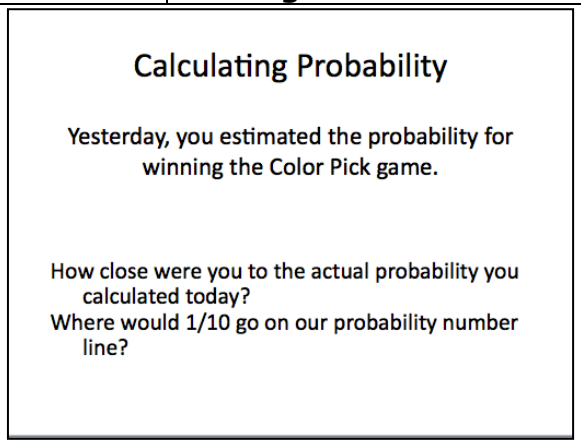
- You can have students identify and group together as a class by moving around the leaves and grouping them. The result should be 10 groups of 6 leaves each.
- *[Alternatively, have students do the grouping first and THEN discuss what happens if when we are choosing a winner.]*



Slide 10

- Students should see that when a trio of colors is drawn for the color pick game, there are 6 distinct "leaves" or outcomes that could be winners out of 60 total outcomes. This can be reduced to 1/10.
- Another way to see this is as 1 of 10 leaf groups. Since we agreed that order does not matter, we might consider each group of leaves as an outcome. In this case, then there is 1 group, or 1 outcome, out of 10 groups, or 10 outcomes, that would be the winner.
- From this illustration, students should realize that it is important to be consistent in thinking about how one is defining outcomes, but as long as consistency is upheld, the answer should still be the same.

12:55	**Closing**



If time, this can launch a mini-discussion on estimating and comparing fractions.

	Exit Ticket/Homework

Slide 12